

# THERE IS NO MONAD BASED ON HARTMAN-MYCIELSKI FUNCTOR

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ABSTRACT. We show that there is no monad based on the normal functor  $H$  introduced earlier by Radul which is a certain functorial compactification of the Hartman-Mycielski construction  $HM$ .

## 0. Introduction

The general theory of functors acting on the category  $Comp$  of compact Hausdorff spaces (compacta) and continuous mappings was founded by Shchepin [Sh]. He described some elementary properties of such functors and defined the notion of the normal functor which has become very fruitful. The classes of all normal functors include many classical constructions: the hyperspace  $exp$ , the space of probability measures  $P$ , the space of idempotent measures  $I$ , and many other functors (cf. [FZ], [TZ], [Z]).

Let  $X$  be a space and  $d$  an admissible metric on  $X$  bounded by 1. By  $HM(X)$  we shall denote the space of all maps from  $[0, 1)$  to the space  $X$  such that  $f|_{[t_i, t_{i+1})} \equiv const$ , for some  $0 = t_0 \leq \dots \leq t_n = 1$ , with respect to the following metric

$$d_{HM}(f, g) = \int_0^1 d(f(t), g(t)) dt, \quad f, g \in HM(X).$$

The construction of  $HM(X)$  is known as the *Hartman-Mycielski construction* [HM] and was introduced for purposes of topological groups theory. However it found some applications not connected with groups (see for example [Z1]).

The construction  $HM$  was considered for any compactum  $Z$  in [TZ; 2.5.2]. Let  $\mathcal{U}$  be the unique uniformity of  $Z$ . For every  $U \in \mathcal{U}$  and  $\varepsilon > 0$ , let

$$\langle \alpha, U, \varepsilon \rangle = \{ \beta \in HM_n(Z) \mid m\{t \in [0, 1) \mid (\alpha(t), \beta(t')) \notin U\} < \varepsilon \}.$$

The sets  $\langle \alpha, U, \varepsilon \rangle$  form a base of a topology in  $HMZ$ . The construction  $HM$  acts also on maps. Given a map  $f : X \rightarrow Y$  in  $Comp$ , define a map  $HM X \rightarrow HM Y$  by the formula  $HMF(\alpha) = f \circ \alpha$ . In general,  $HM X$  is not compact.

Let us fix some  $n \in \mathbb{N}$ . For every compactum  $Z$  consider

$$HM_n(Z) = \left\{ f \in HM(Z) \mid \text{there exist } 0 = t_1 < \dots < t_{n+1} = 1 \right. \\ \left. \text{with } f|_{[t_i, t_{i+1})} \equiv z_i \in Z, i = 1, \dots, n \right\}.$$

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The constructions  $HM_n$  define normal functors in  $Comp$  [TZ; 2.5.2].

Zarichnyi has asked if there exists a normal functor in  $Comp$  which contains all functors  $HM_n$  as subfunctors (see [TZ]). Such a functor  $H$  was constructed in [Ra]. Topological properties of the functor  $H$  were investigated in [RR] and [RR1].

The algebraic aspect of the theory of functors in categories of topological spaces and continuous maps was investigated rather recently. It is based, mainly, on the existence of monad (or triple) structure in the sense of S.Eilenberg and J.Moore [EM]. We recall the definition of monad only for the category  $Comp$ . A monad  $\mathbb{T} = (T, \eta, \mu)$  in the category  $Comp$  consists of an endofunctor  $T : Comp \rightarrow Comp$  and natural transformations  $\eta : Id_{Comp} \rightarrow T$  (unity),  $\mu : T^2 \rightarrow T$  (multiplication) satisfying the relations  $\mu \circ T\eta = \mu \circ \eta T = 1_T$  and  $\mu \circ \mu T = \mu \circ T\mu$ . (By  $Id_{Comp}$  we denote the identity functor on the category  $Comp$  and  $T^2$  is the superposition  $T \circ T$  of  $T$ .)

Many known functors lead to monads: hyperspaces, spaces of probability measures, superextensions etc. There were many investigations of monads in categories of topological spaces and continuous maps (see for example [RZ] or [TZ]). The following question arises naturally: if the functor  $H$  could be completed to a monad? We give a negative answer in this paper.

## 1. Construction of $H$

Let  $X$  be a compactum. By  $CX$  we denote the Banach space of all continuous functions  $\varphi : X \rightarrow \mathbb{R}$  with the usual sup-norm:  $\|\varphi\| = \sup\{|\varphi(x)| \mid x \in X\}$ . We denote the segment  $[0, 1]$  by  $I$ .

For a compactum  $X$  let us define the uniformity of  $HMX$ . For each  $\varphi \in C(X)$  and  $a, b \in [0, 1]$  with  $a < b$  we define a function  $\varphi_{(a,b)} : HMX \rightarrow \mathbb{R}$  by the following formula

$$\varphi_{(a,b)} = \frac{1}{(b-a)} \int_a^b \varphi \circ \alpha(t) dt \text{ for some } \alpha \in HMX.$$

Define

$$S_{HM}(X) = \{\varphi_{(a,b)} \mid \varphi \in C(X) \text{ and } (a,b) \subset (0,1)\}.$$

For  $\varphi_1, \dots, \varphi_n \in S_{HM}(X)$  define a pseudometric  $\rho_{\varphi_1, \dots, \varphi_n}$  on  $HMX$  by the formula

$$\rho_{\varphi_1, \dots, \varphi_n}(f, g) = \max\{|\varphi_i(f) - \varphi_i(g)| \mid i \in \{1, \dots, n\}\},$$

where  $f, g \in HMX$ . The family of pseudometrics

$$\mathcal{P} = \{\rho_{\varphi_1, \dots, \varphi_n} \mid n \in \mathbb{N}, \text{ where } \varphi_1, \dots, \varphi_n \in S_{HM}(X)\},$$

defines a totally bounded uniformity  $\mathcal{U}_{HMX}$  of  $HMX$  (see [Ra]).

For each compactum  $X$  we consider the uniform space  $(HX, \mathcal{U}_{HX})$  which is the completion of  $(HMX, \mathcal{U}_{HMX})$  and the topological space  $HX$  with the topology induced by the uniformity  $\mathcal{U}_{HX}$ . Since  $\mathcal{U}_{HMX}$  is totally bounded, the space  $HX$  is compact.

Let  $f : X \rightarrow Y$  be a continuous map. Define a map  $HMf : HMX \rightarrow HMY$  by the formula  $HMf(\alpha) = f \circ \alpha$ , for all  $\alpha \in HMX$ . It was shown in [Ra] that the map  $HMf : (HMX, \mathcal{U}_{HMX}) \rightarrow (HMY, \mathcal{U}_{HMY})$  is uniformly continuous. Hence there exists a continuous map  $Hf : HX \rightarrow HY$  such that  $Hf|_{HMX} = HMf$ . It is easy to see that  $H : Comp \rightarrow Comp$  is a covariant functor and  $HM_n$  is a subfunctor of  $H$  for each  $n \in \mathbb{N}$ .

Let us remark that the family of functions  $S_{HM}(X)$  embed  $HMX$  in the product of closed intervals  $\prod_{\varphi_{(a,b)} \in S_{HM}(X)} I_{\varphi_{(a,b)}}$  where  $I_{\varphi_{(a,b)}} = [\min_{x \in X} |\varphi(x)|, \max_{x \in X} |\varphi(x)|]$ . Thus, the space  $HX$  is the closure of the image of  $HMX$ . We denote by  $p_{\varphi_{(a,b)}} : HX \rightarrow I_{\varphi_{(a,b)}}$  the restriction of the natural projection. Let us remark that the function  $Hf$  could be defined by the condition  $p_{\varphi_{(a,b)}} \circ Hf = p_{(\varphi \circ f)_{(a,b)}}$  for each  $\varphi_{(a,b)} \in S_{HM}(Y)$ .

We will use some properties of the functor  $H$  proved in [Ra]. Since the functor  $H$  preserves embeddings, we can identify the space  $HA$  with  $Hi(HA) \subset HX$  for each closed subset  $A \subset X$  where  $i : A \rightarrow X$  is the natural embedding. We can define for each  $\alpha \in HX$  the closed set  $\text{supp } \alpha = \cap \{A \text{ is a closed subset of } X \text{ such that } \alpha \in HA\}$ . Since  $H$  preserves preimages, we have  $\alpha \in H(\text{supp } \alpha)$ .

It follows from results of [Ra] and Proposition 5.6 from [Fe] that there exists a unique natural transformation  $\eta : \text{Id}_{\mathcal{C}_{omp}} \rightarrow H$  defined as follows  $\eta X(x)(t) = x$  for each  $t \in [0, 1)$ , where  $x \in X$ . In other words we have  $p_{\varphi_{(a,b)}} \circ \eta X = \varphi$  for each  $\varphi \in C(X)$  and  $(a, b) \subset (0, 1)$ .

## 2. Some technical results

It is easy to check that for each  $\varphi_1, \varphi_2 \in C(X)$ ,  $(a, b) \subset (0, 1)$ ,  $\gamma \in HMX \subset HX$  and  $\lambda_1, \lambda_2 \in \mathbb{R}$  we have  $p_{(\lambda_1 \varphi_1 + \lambda_2 \varphi_2)_{(a,b)}}(\gamma) = \lambda_1 p_{\varphi_1_{(a,b)}}(\gamma) + \lambda_2 p_{\varphi_2_{(a,b)}}(\gamma)$ . As well, if  $\varphi_1 \leq \varphi_2$ , we obtain  $p_{\varphi_1_{(a,b)}}(\gamma) \leq p_{\varphi_2_{(a,b)}}(\gamma)$ . Since  $HMX$  is dense in  $HX$ , we obtain the following two lemmas.

**Lemma 2.1.** *For each  $\varphi_1, \varphi_2 \in C(X)$ ,  $(a, b) \subset (0, 1)$ ,  $\gamma \in HX$  and  $\lambda_1, \lambda_2 \in \mathbb{R}$  we have  $p_{(\lambda_1 \varphi_1 + \lambda_2 \varphi_2)_{(a,b)}}(\gamma) = \lambda_1 p_{\varphi_1_{(a,b)}}(\gamma) + \lambda_2 p_{\varphi_2_{(a,b)}}(\gamma)$ .*

**Lemma 2.2.** *For each  $\varphi_1, \varphi_2 \in C(X)$ ,  $(a, b) \subset (0, 1)$ ,  $\gamma \in HX$  such that  $\varphi_1 \leq \varphi_2$  we have  $p_{\varphi_1_{(a,b)}}(\gamma) \leq p_{\varphi_2_{(a,b)}}(\gamma)$ .*

**Lemma 2.3.** *Consider any  $\nu \in HX$  and a closed subset  $B \subset X$ . Then  $\nu \in HB$  iff  $p_{\varphi_1_{(a,b)}}(\nu) = p_{\varphi_2_{(a,b)}}(\nu)$  for each  $(a, b) \subset (0, 1)$  and  $\varphi_1, \varphi_2 \in C(X)$  such that  $\varphi_1|_B = \varphi_2|_B$ .*

*Proof.* Necessity. The inclusion  $\nu \in HB \subset HX$  means that there exists  $\nu_0 \in HB$  with  $H(i)(\nu_0) = \nu$ , where  $i : B \rightarrow X$  is the natural embedding. Hence, for each  $(a, b) \subset (0, 1)$  and  $\varphi_1, \varphi_2 \in C(X)$  such that  $\varphi_1|_B = \varphi_2|_B$  we have  $p_{\varphi_1_{(a,b)}}(\nu) = p_{\varphi_1 \circ i_{(a,b)}}(\nu_0) = p_{\varphi_2 \circ i_{(a,b)}}(\nu_0) = p_{\varphi_2_{(a,b)}}(\nu)$ .

Sufficiency. We can find an embedding  $j : B \hookrightarrow Y$ , where  $Y \in AR$ . Define  $Z$  to be the quotient space of the disjoint union  $X \cup Y$  obtained by attaching  $X$  and  $Y$  by  $B$ . Denote by  $r : Z \rightarrow Y$  a retraction mapping.

Now take any  $\nu \in HX \subset HZ$  with the property  $p_{\varphi_1_{(a,b)}}(\nu) = p_{\varphi_2_{(a,b)}}(\nu)$  for each  $(a, b) \subset (0, 1)$  and  $\varphi_1, \varphi_2 \in C(X)$  such that  $\varphi_1|_B = \varphi_2|_B$ . We claim that  $H(r)(\nu) = \nu$ . Indeed, take any  $\varphi \in C(Z)$ . Then  $p_{\varphi_{(a,b)}}(H(r)(\nu)) = p_{\varphi \circ r_{(a,b)}}(\nu) = p_{\varphi_{(a,b)}}(\nu)$  since  $\varphi \circ r|_Y = \varphi|_Y$ . Hence,  $\nu \in HX \cap HY = HB$ .

Lemmas 2.1 and 2.3 imply the next lemma.

**Lemma 2.4.** *Consider any  $\nu \in HX$  and a closed subset  $B \subset X$ . Then  $\nu \in FB$  iff  $p_{\varphi_{(a,b)}}(\nu) = 0$  for each  $(a, b) \subset (0, 1)$  and  $\varphi \in C(X)$  such that  $\varphi|_B \equiv 0$ .*

**Lemma 2.5.** *Consider any  $\nu \in HX$  and  $x \in X$ . Then  $x \in \text{supp } \nu$  iff for each neighborhood  $O$  of  $x$  there exists  $a > 0$  such that  $p_{\psi_{(0,1)}}(\nu) \geq a$  for each  $\psi \in C(X, [0, 1])$  such that  $\psi|_O \equiv 1$ .*

*Proof.* Necessity. Suppose the contrary: there exists a neighborhood  $O$  of  $x$  such that for each  $a > 0$  there exists  $\psi \in C(X, [0, 1])$  such that  $\psi|_O \equiv 1$  and  $p_{\psi_{(0,1)}}(\nu) < a$ . Choose any function  $\varphi \in C(X)$  such that  $\varphi|_{X \setminus O} \equiv 0$ . Let  $|\varphi| \leq M > 0$ . By our supposition for each  $\varepsilon > 0$  we can choose  $\psi \in C(X, [0, 1])$  such that  $\psi|_O \equiv 1$  and  $p_{\psi_{(0,1)}}(\nu) < \frac{\varepsilon}{M}$ . Since  $|\varphi| \leq M\psi$ , we obtain  $p_{|\varphi|_{(0,1)}}(\nu) \leq p_{M\psi_{(0,1)}}(\nu) < \varepsilon$  using Lemmas 2.1 and 2.2. Thus we have  $p_{|\varphi|_{(0,1)}}(\nu) < \varepsilon$  for each  $\varepsilon > 0$ , hence  $p_{|\varphi|_{(0,1)}}(\nu) = 0$ . It is easy to check that  $p_{\varphi_{(0,1)}}(\nu) = 0$  too. Then we have  $\nu \in H(X \setminus O)$ , hence  $x \notin \text{supp } \nu$ .

Sufficiency. Suppose  $x \notin \text{supp } \nu$ . Choose a neighborhood  $O$  of  $x$  such that  $\text{Cl } O \cap \text{supp } \nu = \emptyset$ . There exists a function  $\psi \in C(X, [0, 1])$  such that  $\psi|_O \equiv 1$  and  $\psi|_{\text{supp } \nu} \equiv 0$ . Then  $p_{\psi_{(0,1)}}(\nu) = 0$  by Lemma 2.4. Thus, we obtain a contradiction and the lemma is proved.

### 3. The main result

For any natural number  $n \in \mathbb{N}$  by  $K_n$  we denote the finite compactum  $\{1, \dots, n\}$  (with discrete topology). Define  $\alpha \in HM(K_n \times K_n) \subset H(K_n \times K_n)$  and  $\beta \in HM(K_n) \subset H(K_n)$  as follows  $\alpha(s) = (i; i)$  and  $\beta(s) = i$  if  $s \in [\frac{i-1}{n}, \frac{i}{n})$  for  $i \in K_n$ ,  $s \in [0, 1)$ . By  $pr_l : K_n \times K_n \rightarrow K_n$  for  $l \in \{1, 2\}$  we denote the natural projections.

**Lemma 3.1.** *We have  $(H(pr_1))^{-1}(\beta) \cap (H(pr_2))^{-1}(\beta) = \{\alpha\}$ .*

*Proof.* Consider any  $\gamma \in (H(pr_1))^{-1}(\beta) \cap (H(pr_2))^{-1}(\beta)$ . Firstly, let us show that  $\text{supp } \gamma \subset \text{supp } \alpha = \{(i; i) | i \in K_n\}$ . Suppose the contrary. Then there exist  $i, j \in K_n$  such that  $i \neq j$  and  $(i; j) \in \text{supp } \gamma$ . Consider a function  $\psi : K_n \times K_n \rightarrow [0, 1]$  such that  $\psi(i; j) = 1$  and  $\psi(k; l) = 0$  for each  $(k; l) \neq (i; j)$ . By Lemma 2.5 there exists  $a > 0$  such that  $p_{\psi_{(0,1)}}(\gamma) \geq a$ . For  $r \in K_n$  define a function  $\varphi_r : K_n \rightarrow \mathbb{R}$  by the formula  $\varphi_r(s) = 1$  if  $r = s$  and  $\varphi_r(s) = 0$  if  $r \neq s$ . For  $k \in \{1, 2\}$  and  $r \in K_n$  we consider the functions  $\varphi_r^k = \varphi_r \circ pr_k : K_n \times K_n \rightarrow \mathbb{R}$ . Choose a neighborhood  $V$  of  $\gamma$  defined as follows  $V = \{\gamma' \in H(K_n \times K_n) \mid |p_{\psi_{(0,1)}}(\gamma) - p_{\psi_{(0,1)}}(\gamma')| < \frac{a}{2} \text{ and } |p_{\varphi_r^k}(\gamma) - p_{\varphi_r^k}(\gamma')| < \frac{a}{2n} \text{ for each } k \in \{1, 2\} \text{ and } r \in K_n\}$ .

Consider any  $\gamma_1 \in HMX \cap V$ . Since  $|p_{\psi_{(0,1)}}(\gamma) - p_{\psi_{(0,1)}}(\gamma_1)| < \frac{a}{2}$ , we have  $m\{t \in [0, 1) \mid \gamma_1(t) = (i; j)\} > \frac{a}{2}$ . Hence there exists  $r \in \{1, \dots, n\}$  such that  $m\{t \in [\frac{r-1}{n}, \frac{r}{n}) \mid \gamma_1(t) = (i; j)\} > \frac{a}{2n}$ .

If  $r \neq i$  we have  $p_{\varphi_r^1}(\gamma_1) = n \int_{\frac{r-1}{n}}^{\frac{r}{n}} \varphi_r^1 \circ \gamma_1(t) dt < 1 - \frac{a}{2}$ . But  $p_{\varphi_r^1}(\gamma) = p_{\varphi_r^1 \circ pr_1}(\gamma) = p_{\varphi_r^1 \circ H(pr_1)}(\gamma) = p_{\varphi_r^1}(\beta) = 1$  and we obtain a contradiction with the definition of  $V$ .

If  $r = i$ , then we have  $r \neq j$  and we obtain a contradiction using similar arguments for the second projection  $pr_2$  and the function  $\varphi_r^2$ . Hence we have the inclusion  $\text{supp } \gamma \subset \text{supp } \alpha$ .

Consider any  $\varphi \in C(K_n \times K_n)$  and  $(a, b) \subset (0, 1)$ . Define  $\psi \in C(K_n)$  as follows  $\psi(i) = \varphi(i; i)$  for  $i \in K_n$  and put  $\xi = \psi \circ pr_1$ . Since  $\text{supp } \gamma \subset \{(i; i) | i \in K_n\}$ , we have  $p_{\varphi_{(a,b)}}(\gamma) = p_{\xi_{(a,b)}}(\gamma)$  by Lemma 2.3. Then  $p_{\varphi_{(a,b)}}(\gamma) = p_{\psi \circ pr_1(a,b)}(\gamma) = p_{\psi(a,b)}(\beta) = p_{\psi \circ pr_1(a,b)}(\alpha) = p_{\varphi_{(a,b)}}(\alpha)$ . Hence  $\alpha = \gamma$ .

**Theorem 3.2.** *There is no natural transformation  $\mu : H^2 \rightarrow H$  such that  $\mu \circ H\eta = \mu \circ \eta H = \mathbf{1}_H$ .*

*Proof.* Suppose that there exists such natural transformation. Let  $n \in \mathbb{N}$ . For  $i \in K_n$  define  $\alpha_i \in HM(K_n \times K_n) \subset H(K_n \times K_n)$  as follows  $\alpha_i(s) = (i; j)$  if  $s \in [\frac{i-1}{n}, \frac{i}{n})$  for  $j \in K_n$  and  $s \in [0, 1)$ . We also define  $\mathcal{A}_n \in HM^2(K_n \times K_n) \subset$

$H^2(K_n \times K_n)$  as follows  $\mathcal{A}_n(s) = \alpha_i$  if  $s \in [\frac{i-1}{n}, \frac{i}{n})$  for  $i \in K_n$  and  $s \in [0, 1)$ . Put  $H^2(pr_l)(\mathcal{A}_n) = \mathcal{C}_l$  for  $l \in \{1, 2\}$ . Then we have  $\mathcal{C}_1 = H(\eta K_n)(\beta)$  and  $\mathcal{C}_2 = \eta H K_n(\beta)$ . Hence  $\mu K_n(\mathcal{C}_1) = \mu K_n(\mathcal{C}_2) = \beta$ . Since  $\mu$  is a natural transformation, we have  $\mu K_n \times K_n(\mathcal{A}_n) \in (H(pr_1))^{-1}(\beta) \cap (H(pr_2))^{-1}(\beta)$ . Hence we obtain  $\mu K_n \times K_n(\mathcal{A}_n) = \alpha$  by previous lemma.

By  $D$  we denote the two-point set  $\{0, 1\}$  with discrete topology. For  $i \in \{1, \dots, n\}$  define  $\gamma_i \in HMD \subset HD$  as follows  $\gamma_i(s) = 1$  if  $s \in [\frac{i-1}{n}, \frac{i}{n})$  and  $\gamma_i(s) = 0$  otherwise for  $s \in [0, 1)$ . We also define  $\mathcal{B}_n \in HM^2D \subset H^2D$  by conditions  $\mathcal{B}_n(s) = \gamma_i$  if  $s \in [\frac{i-1}{n}, \frac{i}{n})$  for  $i \in \{1, \dots, n\}$  and  $s \in [0, 1)$ . Consider a map  $f : K_n \times K_n \rightarrow D$  defined as follows  $f(i; j) = 1$  if  $i = j$  and  $f(i; j) = 0$  otherwise. It is easy to see that  $Hf(\mathcal{A}_n) = \mathcal{B}_n$ . Since  $\mu$  is a natural transformation, we have  $\mu D(\mathcal{B}_n) = Hf \circ \mu K_n \times K_n(\mathcal{A}_n) = Hf(\alpha) = \eta D(1)$ . But it is easy to see that  $\mathcal{B}_n$  converges to  $\eta HD(\eta D(0))$  if  $n \rightarrow \infty$ . Hence  $\mu D$  is not continuous and we obtain a contradiction.

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